1. **Asymptotes, but Backwards.** Find a rational function that has simultaneously a horizontal asymptote at $y = 1$, vertical asymptotes at $x = 2$ and at $x = -2$ and a removable discontinuity at $x = 0$.

Is it possible for a rational function to have more than 1 horizontal asymptote? Can an algebraic function have more than 1 horizontal asymptote? More than 2?

2. **Sequences.** A sequence is an infinite ordered list of numbers defined by a function whose inputs are positive integers. For example, the positive even integers form a sequence

$$\{2, 4, 6, 8, \dots\}$$

and is defined by the function $f(n) = 2n$, where $n = 1, 2, 3, 4, \dots$
The limit of a sequence (if it exists) is $\lim_{n\to\infty} f(n)$. *All limit laws that apply to limits at infinity apply to limits of sequences.*

Find a function to describe the following sequences and find the limit of the sequence:

(a) $\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\}$

(b) $\{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}\}$

(c) $\{4, 2, \frac{4}{3}, 1, \frac{4}{5}, \frac{2}{3}, \dots\}$

3. **Trig-*ish* Functions.** The *hyperbolic functions* are another class of elementary functions. The *hyperbolic sine*, $\sinh x$ usually read as "cinch of x" and *hyperbolic cosine*, $\cosh x$ are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Determine the end behavior of both functions and decide for what values of x the functions are continuous.

4. **Continuity of Piecewise Functions** Find values of a and b so that

$$f(x) = \begin{cases} x^2 & \text{if } x < -b \\ ax + 2 & \text{if } -b \leq x \leq b \\ -x^2 + 4 & \text{if } x > b \end{cases}$$

is continuous for all real numbers x.